Developmental Guidelines
## Developmental Guidelines

### Number Development – Whole Numbers

<table>
<thead>
<tr>
<th>MMSD Standards</th>
<th>Common Core</th>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-K</td>
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</tbody>
</table>

#### Forward Number Word Sequence
- May be able to count to 10 or beyond but does not have 1:1 correspondence consistently
- Can count 1-20
- Can name the number directly after for numbers 1-20 but may have to count from 1 to do so.
- Can count by 1s from any number 1-30;
- Can name number directly after for numbers 1-30

#### Backward Number Word Sequence
- Can count back from 10;
- Can name the number directly before for numbers 1-5 but may have to count from 1
- Can consistently count back within 10-0 (starting from any number);
- Can inconsistently count back from 30;
- Can inconsistently name the number directly before for numbers from 1-30 but may have to count from smaller number;

#### Sequencing/Ordering
- Can sequence numbers in the range 1-20
- Can sequence numbers in the range 1-30
- Can sequence and order numbers in the range 1-100
- Can sequence and order numbers in the range 1-200

#### Number Line/Magnitude
- Can accurately place numbers 1-20 on an empty number line
  (Example: Where does 15 go?)

#### Subitizing
- Can subitize regular dot patterns to 6
- Can subitize finger patterns to 5
- Can subitize quantities in pair-wise and five-wise ten frames
- Can subitize finger patterns to 10

#### Object Counting
- Can count or make sets of up to 10 accurately.
- Can count or make sets of up to 30 accurately.
- Can count or make sets of up to 100 using base-ten materials (10s and 1s)
# Developmental Guidelines

## Number Development – Whole Numbers

<table>
<thead>
<tr>
<th></th>
<th>Level 5</th>
<th>Level 6</th>
<th>Levels 7-9</th>
<th>Level 10+</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MMUSD Standards</strong></td>
<td>2</td>
<td>2/3</td>
<td>3/4</td>
<td>5+</td>
</tr>
<tr>
<td><strong>Common Core</strong></td>
<td>1/2</td>
<td>3/4</td>
<td>4</td>
<td>4+</td>
</tr>
<tr>
<td><strong>Forward Number Word Sequence</strong></td>
<td></td>
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</tr>
<tr>
<td>Can count by 1s from any number 1-1000; Can name the number directly after for numbers 1-1000</td>
<td></td>
<td></td>
<td>Can count by 1s from any number 1-10,000 Can name the number directly after for numbers 1-10,000</td>
<td>Can count forward and backward by 1s, 10s, 100s, 1,000s and 10,000s from any number up to 100,000</td>
</tr>
<tr>
<td>Can count back within 1000-0 (starting from any number); Can name the number directly before for numbers 1-1,000</td>
<td></td>
<td></td>
<td>Can count back within 10,000-0 (starting from any number); Can name the number directly before for numbers 1-10,000</td>
<td>Can count forward and backward by 1s, 10s, 100s, 1,000s and 10,000s from any number up to 100,000</td>
</tr>
<tr>
<td><strong>Backward Number Word Sequence</strong></td>
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</tr>
<tr>
<td>Can count by 2s, 3s, 4s, and 5s starting from any multiple Can count forward and backward by 10s from any 2-digit number</td>
<td></td>
<td></td>
<td>Can count forward and backward by 10s and 100s from any 2 or 3 digit number Can count forward and backward by 25s starting from any multiple</td>
<td>Can count forward and backward by 1s, 10s, 100s, 1,000s and 10,000s from any number up to 100,000</td>
</tr>
<tr>
<td>Can count back within 1000-0 (starting from any number); Can name the number directly before for numbers 1-1,000</td>
<td></td>
<td></td>
<td>Can count back within 10,000-0 (starting from any number); Can name the number directly before for numbers 1-10,000</td>
<td>Can count forward and backward by 1s, 10s, 100s, 1,000s and 10,000s from any number up to 100,000</td>
</tr>
<tr>
<td><strong>Number Word Sequences: Skip Counting</strong></td>
<td></td>
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<td></td>
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<tr>
<td>Can identify and write numerals 1-1000</td>
<td></td>
<td></td>
<td>Can identify and write numerals 1-10,000</td>
<td>Can identify and write numerals beyond 100,000</td>
</tr>
<tr>
<td>Can identify and write numerals in the range 1-1000</td>
<td></td>
<td></td>
<td>Can sequence and order numbers in the range 1-10,000</td>
<td>Can sequence and order numbers beyond 100,000</td>
</tr>
<tr>
<td>Can accurately place numbers 1-100 on an empty or nearly empty number line (Example: Where does 65 go?)</td>
<td></td>
<td></td>
<td>Can accurately place numbers 1-1000 on an empty or nearly empty number line (Example: Where does 215 go?)</td>
<td>Can accurately place whole numbers on empty or nearly empty number lines.</td>
</tr>
<tr>
<td><strong>Number Line/ Magnitude</strong></td>
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</tr>
<tr>
<td>Can count or make sets of up to 1,000 using base-ten materials (100s, 10s, and 1s)</td>
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<tr>
<td><strong>Object Counting</strong></td>
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</tbody>
</table>
# Developmental Guidelines

## Operation Development – Addition and Subtraction with whole numbers

<table>
<thead>
<tr>
<th>MMSSD Standards</th>
<th>Common Core</th>
<th>Problem Types</th>
<th>Strategies Supported by Materials</th>
<th>Counting Strategies</th>
<th>Number Relationship Strategies</th>
<th>Composing/Decomposing</th>
<th>Fact Fluency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Solves problems embedded in daily routines using direct modeling including JRU and SRU contexts with totals to 5</td>
<td>Can model problems using concrete objects, and count the objects to find the answer. Involving the child in acting out the problem may support the child’s understanding of the problem.</td>
<td>When solving joining problems, counts from 1, but may continue the count when counting the second set. (i.e., may count the first set from 1, then the second set from 1, then the joined set from 1; or may count the first set from 1, then continue by counting the second set to reach the total). When solving separating problems, counts forward three times (counts the starting quantity, then the items to be removed, then the remaining items).</td>
<td>Mentally: adds 10 to a single-digit number adds a single-digit number to any decade number subtracts to a decade, e.g., 56 - 6 = 50. adds up to a decade, e.g., 48 + 50 subtracts a single-digit number from a decade number, e.g., 60 - 4 = 56. Applies an informal understanding of: the commutative and associative properties of addition (i.e., does not always add numbers in the order in which they are presented.) the relationship between addition and subtraction (e.g., 12 - 9 can also be thought of as 9 + 3 = 12)</td>
<td>Can mentally compose and decompose numbers to 5 Can conceptually subitize quantities to 5</td>
<td>Fluent with addition and subtraction facts to 5, doubles to 5+5, and +1 facts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Uses strategies described below to solve problems embedded in daily routines using direct modeling including JRU and SRU contexts with totals to 10</td>
<td>Can solve a covered task when one or both sets are covered, the first number is within 1-20, the second number is within 1-5, and the total is within 1-20. Counts from 1 to do so.</td>
<td>Counts on 1, 2 or 3 from the first addend within 30. Counts back 1, 2, or 3 when minuend is 10 or fewer.</td>
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<tr>
<td></td>
<td></td>
<td>Uses strategies described below to solve JRU and SRU story problems with totals to 20 using direct modeling</td>
<td>Can count on to solve additive (e.g., 15 + 2 = ) and missing addend (e.g., 15 + = 17) covered tasks Can count back to solve removed item covered tasks (e.g., 17 – 2 = ) May be able to count up or back to solve missing subtrahend covered tasks (e.g., 17 – = 15)</td>
<td>Counts on when the first addend is within 1-100 and the second addend is 1-5. Counts back 1-5 when the starting number is within 1-100. Can count up or back 1-5 to find the difference between two numbers 1-100 (e.g., 46 – 43)</td>
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<tr>
<td></td>
<td></td>
<td>Uses strategies described below to solve story problems with totals to at least 30, including: JRU, SRU, JCU, SCU, CDU, PPW-WU</td>
<td>Uses strategies described below to solve story problems with totals to at least 30, including: JRU, SRU, JCU, SCU, CDU, PPW-WU</td>
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</tr>
<tr>
<td>Level 0</td>
<td>Pre-K</td>
<td>Level 1</td>
<td>K</td>
<td>Level 2</td>
<td>K/1</td>
<td>Level 3</td>
<td>1/2</td>
</tr>
<tr>
<td>Level 1</td>
<td>Pre-K/K</td>
<td>Level 2</td>
<td>K</td>
<td>Level 3</td>
<td>K/1</td>
<td>Level 4</td>
<td>1/2</td>
</tr>
<tr>
<td>Level 2</td>
<td>K</td>
<td>Level 3</td>
<td>K/1</td>
<td>Level 4</td>
<td>Uses strategies described below to solve story problems with totals to 100, including: JRU, SRU, JCU, SCU, CDU, PPW-WU</td>
<td></td>
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<tr>
<td>Level 3</td>
<td>K/1</td>
<td>Level 4</td>
<td>Uses strategies described below to solve story problems with totals to 100, including: JRU, SRU, JCU, SCU, CDU, PPW-WU</td>
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</tr>
</tbody>
</table>
### Developmental Guidelines

#### Operation Development – Addition and Subtraction Cont.

<table>
<thead>
<tr>
<th>MMSD Standards</th>
<th>Common Core</th>
<th>Level 5</th>
<th>Level 6</th>
<th>Level 7</th>
<th>Level 8+</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
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<td>2/3</td>
<td>2/3</td>
<td>3</td>
<td>3/4+</td>
</tr>
</tbody>
</table>

**Problem Types**

- Uses strategies described below to solve story problems including: JRU, SRU, JCU, CDU, SCU, PPW-WU, PPW-PU
  (Appropriate number choices are described below.)

**Strategies Supported by Materials**

- Can solve additive and subtractive covered tasks involving two-digit numbers modeled with base-10 materials

- Can solve additive and subtractive covered tasks involving three-digit numbers when supported by base-10 materials

**Counting Strategies**

- Mentally:
  - Adds 10 or subtracts 10 from any number 10-100.
  - Adds any decade to or subtracts any decade from numbers 10-100: e.g. 20 + 40, 56 + 30, 72 – 40.
  - Uses addition facts such as 4 + 3 to solve problems such as 54 + 3 and 40 + 30, and subtraction facts such as 7 - 5 to solve 57 - 5 and 70 - 50.
  - Decomposes a single-digit number to cross a decade when adding or subtracting within 100 e.g. 48 + 6 = (48 + 2) + 4 or 56 – 9 = (56 – 6) – 3

- Has at least one viable strategy for solving addition and subtraction problems involving any 2-digit numbers (such as incrementing, tens-and-ones, or compensating).
  - Can write down intermediate steps when needed to keep track of thinking.

- Mentally:
  - Adds 100 (or 200, 300, etc.) to or subtracts 100 (or 200, 300, etc.) from any three-digit number 100-900 (e.g., 438 + 100, 350 – 200), finds the other part of 100 (e.g. 88 + 7 = 100).
  - Uses addition facts such as 7 + 8 to solve problems such as 157 + 8 or 70 + 80, and subtraction facts such as 14 – 9 to solve problems such as 314 – 9 or 140 – 90.
  - Decomposes a single-digit number to cross a hundred when adding or subtracting within 1,000 e.g., 497 + 5 = (497 + 3) + 2
  - Applies an informal understanding of:
    - The commutative and associative properties of addition
    - The relationship between addition and subtraction

**Number Relationship Strategies**

- Applies an informal understanding of:
  - The commutative and associative properties of addition
  - The relationship between addition and subtraction

**Computing- decomposing**

- Can mentally compose and decompose numbers to 100 into tens and ones in standard and non-standard ways (e.g. 74 is 70 + 4, 60 + 14, and 50 + 24)

- Can mentally compose and decompose numbers to 1,000 in to hundreds, tens, and ones in standard and non-standard ways (e.g. 247 is 200 + 40 + 7, and 100 + 130 + 17)

**Fad Fluency**

- Fluent with addition facts (with totals to 20)
- Fluent with subtraction facts (with totals to 10)
- Fluent with some across-ten subtraction facts
- Fluent with subtraction facts (with totals to 20)
# Developmental Guidelines

## Operation Development – Multiplication and Division with whole numbers

<table>
<thead>
<tr>
<th>Problem Types</th>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solves</td>
<td>Pre-K</td>
<td>Pre-K/K/1</td>
<td>K/1</td>
<td>K/1/2</td>
<td>1/2/3</td>
</tr>
</tbody>
</table>

**MMSSD Standards**

<table>
<thead>
<tr>
<th>Common Core</th>
<th>Multiplication is not addressed in the Common Core State Standards before grade 2.</th>
</tr>
</thead>
</table>

### Problem Types

- **Solves Multiplication, Measurement Division, and Partitive Division problems with totals to 5 embedded in daily routines using direct modeling.**
- **Uses strategies described below to solve Multiplication, Measurement Division, and Partitive Division story problems with totals to 20.**
- **Uses strategies described below to solve Multiplication, Measurement Division, and Partitive Division story problems with totals to 30.**
- **Uses strategies described below to solve Multiplication, Measurement Division, and Partitive Division story problems with totals to 50.**

### Strategies Based on Visual Models

- **Can describe, organize, and make equal groups and equal shares when supported by concrete objects.**
- **Objects at this level need to be fairly literal representations of the quantities in the problem (e.g. coats and buttons, plates and cookies, etc.) to help children differentiate between objects that represent the number of groups, and objects that represent the items in each group.**
- **Involving the child in acting out the problem may support the child’s understanding of the problem.**

### Direct Modeling and Counting Strategies

- **Counts by ones to determine answers, and does not pay attention to the structure of groups when counting.**
  - (In other words, once the child has modeled the problem, he or she sees all the items to be counted as a large collection of ones, rather than as several equal groups. Depending on the model that the child has created, he or she may or may not count one group, and then another group, and then another group. A child at this level who is trying to find out how many squares or dots are in a rectangular array will probably not count each row, or each column—he or she may count the items in a zigzag or spiral path.)

### Additive Strategies

- **May use repeated addition to solve multiplication problems.**
  - For example: for 6×4, the child might draw:
    - May use repeated addition to solve multiplication problems.
    - (Developmental Guidelines for Addition and Subtraction at this level show strategies that students are likely to use to find answers when using repeated addition.)
### Developmental Guidelines

**Operation Development – Multiplication and Division Cont.**

<table>
<thead>
<tr>
<th>Level 5</th>
<th>Level 6</th>
<th>Level 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMSD Standards</td>
<td>2/3</td>
<td>2/3</td>
</tr>
<tr>
<td>Common Core</td>
<td>2/3</td>
<td>2/3</td>
</tr>
</tbody>
</table>

#### Problem Types

- **Multiplicative Strategies**
  - Uses strategies described below to solve:
    - Multiplication story problems with factors up to 10×10
    - MD and PD story problems with dividends up to 50 and divisors up to 10. (May be able to solve MD and PD story problems with dividends to 100)

- **Additive Strategies**
  - Can combine two sets of equal groups when each set is covered:
    - Example: There are four rows with four dots in each row. How many dots is that? If I add two more rows of four, how many rows are there? How many dots altogether?

- **Properties**
  - Begins to informally apply the commutative property and distributive property. Note that students use strategies that employ the properties before they fully understand why the properties work.
    - Begins to understand the inverse relationship between multiplication and division.

#### Strategies Based on Visual Models

- **Example:** There are four rows with four dots in each row. How many dots is that? If I add two more rows of four, how many dots altogether?

#### Direct Modeling and Counting Strategies

- **Examples:**
  - **Example:** Four squares like the one shown can fit in a row across the top of this rectangle. Five squares fit down the side. How many squares would it take to cover the whole rectangle?
  - **Example:** If you finish covering this rectangle with squares, how many squares would there be altogether?

#### Additive Strategies

- **Uses known facts:**
  - For 12×6, the child may do: 10×8 = 80, 80+8×8 = 96
  - For 32×8, the child might say, "3×10 is 30, so it’s 3 with 2 left over.

- **Uses strategies such as repeated addition, repeated subtraction, repeated doubling, or other additive groupings to solve multiplication and division problems.**
  - For example, for 96÷16, the child might do:
    - 96÷16 = 6
    - 16×6 = 96
  - **Example:** Four squares like the one shown can fit in a row across the top of this rectangle. Five squares fit down the side. How many squares would it take to cover the whole rectangle?

- **Uses strategies such as repeated addition, repeated subtraction, repeated doubling, or other additive groupings to solve multiplication and division problems.**
  - For example, for 96÷16, the child might do:
    - 96÷16 = 6
    - 16×6 = 96
  - **Example:** Four squares like the one shown can fit in a row across the top of this rectangle. Five squares fit down the side. How many squares would it take to cover the whole rectangle?

#### Multiplicative Strategies

- **Uses strategies such as repeated addition, repeated subtraction, repeated doubling, or other additive groupings to solve multiplication and division problems.**
  - For example, for 96÷16, the child might do:
    - 96÷16 = 6
    - 16×6 = 96
  - **Example:** Four squares like the one shown can fit in a row across the top of this rectangle. Five squares fit down the side. How many squares would it take to cover the whole rectangle?

#### Fact Fluency

- **Fluent with multiplication facts for 0×0, x1, x2, x10, x5 (including, for example, both 8×5 and 5×8)**
- **Fluent with multiplication facts for 0×0, x1, x2, x10, x5, x3, x4**
### Developmental Guidelines

#### Operation Development – Multiplication and Division Cont.

<table>
<thead>
<tr>
<th>MN5O Standards</th>
<th>Common Core</th>
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<tbody>
<tr>
<td>3/4</td>
<td>3/4</td>
</tr>
</tbody>
</table>

**Problem Types**
- Uses strategies described below to solve:
  - Multiplication story and number problems (with products of 1-digit x 2-digit numbers)
  - MD and PD story and number problems (with dividends up to 1,000 where either the divisor or the quotient is a single-digit number)
  - Story problems that involve:
    - multiple steps and multiple operations
    - the interpretation of remainders

**Strategies Based on Visual Models**
- Uses tiling, base-ten blocks, or grid paper to show that 6×15 is 6×10 + 6×5, or 2×15+2×15+2×15:

<table>
<thead>
<tr>
<th>6</th>
<th>10</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6×10=30</td>
<td>6×5=30</td>
</tr>
</tbody>
</table>

- Begins to use open arrays:

<table>
<thead>
<tr>
<th>6</th>
<th>10</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6×10=30</td>
<td>6×5=30</td>
</tr>
</tbody>
</table>

**Additive Strategies**
- Uses strategies such as repeated addition, repeated subtraction, repeated doubling, or other additive groupings to solve multiplication and division problems.

**Multiplicative Strategies**
- Uses partial products for products of 1-digit x 2-digit numbers, especially where the 2-digit number is relatively small. For example, for 15×7:

<table>
<thead>
<tr>
<th>Groups of divisor</th>
<th>Breaking dividend apart</th>
</tr>
</thead>
<tbody>
<tr>
<td>10×7 = 70</td>
<td>200 is 8 groups of 25</td>
</tr>
<tr>
<td>5×7 = 35</td>
<td>You can put 1 more in each group.</td>
</tr>
<tr>
<td>70+35 = 105</td>
<td>So there are 26 in each group with 4 left over.</td>
</tr>
</tbody>
</table>

- Solves division problems by using groups of the divisor to build up to the dividend (especially for Measurement Division), or by breaking the dividend into parts (especially for Partitive Division). For example, for 212÷8:

- Applies an informal understanding of:
  - the commutative property of multiplication
  - the distributive property
  - the relationship between multiplication and division

**Properties**
- Applies an informal understanding of:
  - the commutative and associative properties of multiplication
  - the distributive property
  - the relationship between multiplication and division

**Estimation**
- Judges the reasonableness of results of multiplication problems

**Fact Fluency**
- Fluent with multiplication facts for x0, x1, x2, x10, x5, x3, x4
- Fluent with some multiplication facts for x6, x7, x8, x9
- Fluent with all multiplication facts to 10×10

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**Level 8**

- Uses strategies described below to solve:
  - Multiplication story and number problems (with products of 1-digit x 2-digit numbers)
  - MD and PD story and number problems (with dividends up to 1,000 where either the divisor or the quotient is a single-digit number)
  - Story problems that involve:
    - multiple steps and multiple operations
    - the interpretation of remainders

**Strategies Based on Visual Models**
- Uses open arrays to model multi-digit problems. For example, for 8×47:

<table>
<thead>
<tr>
<th>8</th>
<th>40</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>8×40 = 320</td>
<td>8×7 = 56</td>
<td></td>
</tr>
</tbody>
</table>

**Additive Strategies**
- Mentally multiplies one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9×80, 5×60)

**Multiplicative Strategies**
- Can flexibly choose among strategies such as partial products, making an equivalent problem (doubling and halving), or compensating to solve products of 1-digit x 2-digit and 1-digit x 3-digit numbers efficiently and accurately. For example:

- Making an equivalent problem (Doubling and halving)

  - For 35×8:
    - 35×8 = 70×4
    - = 140+2
    - = 280

- Solves division problems by using groups of the divisor to build up to the dividend, breaking the dividend into parts, making an equivalent problem, or compensating. For example:

- Making an equivalent problem

  - For 168÷24 = 84÷12
    - = 42÷6
    - = 21÷3
    - = 7

**Fact Fluency**
- Fluent with multiplication facts for x0, x1, x2, x10, x5, x3, x4
- Fluent with some multiplication facts for x6, x7, x8, x9
- Fluent with all multiplication facts to 10×10

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**Level 9**

- Uses strategies described below to solve:
  - Multiplication story and number problems (with products of 1-digit x 2-digit numbers and 1-digit x 3-digit numbers)
  - MD and PD story and number problems (with dividends up to 1,000 where either the divisor or the quotient is a single-digit number)
  - Story problems that involve:
    - multiple steps and multiple operations
    - the interpretation of remainders

**Strategies Based on Visual Models**
- Uses 3-D arrays to explore the associative property. For example, for 126 = (6×7)×3 = 6×(7×3)

**Additive Strategies**
- Mentally multiplies one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9×80, 5×60)

**Multiplicative Strategies**
- Can flexibly choose among strategies such as partial products, making an equivalent problem (doubling and halving), or compensating to solve products of 1-digit x 2-digit and 1-digit x 3-digit numbers efficiently and accurately. For example:

- Making an equivalent problem (Doubling and halving)

  - For 35×8:
    - 35×8 = 70×4
    - = 140+2
    - = 280

- Solves division problems by using groups of the divisor to build up to the dividend, breaking the dividend into parts, making an equivalent problem, or compensating. For example:

- Making an equivalent problem

  - For 235÷80:
    - 3×80 = 240, so 3 is too much.
    - The answer is 2 with 75 left over.
## Developmental Guidelines

### Operation Development – Multiplication and Division Cont.

<table>
<thead>
<tr>
<th>MMSD Standards</th>
<th>Level 10</th>
<th>Level 11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Core</td>
<td>4/5/6</td>
<td>5/6</td>
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</tbody>
</table>

### Problem Types

**Strategies Based on Visual Models**

- Uses open arrays to model multi-digit problems. For example, for 32×48:
  
<table>
<thead>
<tr>
<th>2</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30×40 = 1,200</td>
</tr>
<tr>
<td>4</td>
<td>2×40 = 80</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

- Mentally solves extended multiplication facts (e.g., 3×500, 8,000×7, 40×60)
  - Can **flexibly** choose among strategies such as partial products, making equivalent problems, compensating, and ratio tables to solve products of 2-digit × 3-digit numbers up to 4 digits **efficiently and accurately**.
  - Uses strategies such as partial products, making equivalent problems, compensating, and ratio tables to solve products of 2-digit × 3-digit numbers.

- Solves division problems by using groups of the divisor to build up to the dividend, breaking the dividend into parts, making an equivalent problem, or compensating.

- Applies an informal understanding of:
  - The commutative and associative properties of multiplication
  - The distributive property
  - The relationship between multiplication and division

- Judges the reasonableness of results of multiplication problems

### Composing/Decomposing

- Can:
  - Find all factor pairs for a whole number in the range 1-100
  - Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number.
  - Determine whether a given whole number in the range 1-100 is prime or composite.
  - Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognizes that 700 + 70 = 10 by applying concepts of place value and division.

### Fact Fluency

- Fluent with some division facts
- Fluent with all division facts

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**Draft: August 5, 2011**
# Operation Development – Multiplication and Division Cont.

## Level 12+

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<tr>
<th>MMSD Standards</th>
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<td>Common Core</td>
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</tbody>
</table>

### Problem Types

Uses strategies described below to solve:
- Multiplication, MD and PD story and number problems with totals to 10,000 and beyond
- Story problems that involve:
  - multiple steps and multiple operations
  - multiplicative comparisons
  - rates, proportions, ratios
  - combinations
  - the interpretation of remainders

### Multiplicative Strategies

- Can **flexibly** choose among strategies such as partial products, making equivalent problems, compensating, ratio tables, and other algorithms to solve products of 2-digit × 3-digit numbers **efficiently** and **accurately**.
- Can **flexibly** choose among strategies such as using groups of the divisor to build up to the dividend, breaking the dividend into parts, making an equivalent problem or compensating to **efficiently** and **accurately** solve division problems involving dividends up to 1,000 and divisors up to 100.

  (For multiplication and division problems involving numbers larger than these, the most efficient strategy may involve estimating the magnitude of the answer and using a calculator to perform the computation.)

### Properties

- Applies an informal understanding of:
  - the commutative and associative properties of multiplication
  - the distributive property
  - the relationship between multiplication and division

### Estimation

- Judges the reasonableness of results of multiplication and division problems

### Composing/Decomposing

- Can find the prime factorization for numbers in the range 1-100
- Can express the prime factorization of numbers using exponential notation.
- Can generalize about patterns in the number of zeros of the product when multiplying numbers involving powers of 10. Uses whole-number exponents to denote powers of 10.
Works Consulted


